

$$\begin{aligned} \textcircled{1} \quad & A(2; 4; 6) \\ & B(-3; -5; -1) \\ & C = \frac{1}{2} AB \\ \hline \text{a) } & C(x; y; z) \\ \text{б) } & |\overrightarrow{AB}| \end{aligned}$$

$$\text{a) } \overrightarrow{AB} = (-3 - 2; -5 - 4; -1 - 6) = (-5; -9; -7);$$

$$\vec{C} = \frac{1}{2} \overrightarrow{AB} = (-2,5; -4,5; -3,5).$$

$$\begin{aligned} \text{б) } |\overrightarrow{AB}| &= \sqrt{(-5)^2 + (-9)^2 + (-7)^2} = \\ &= \sqrt{25 + 81 + 49} = \sqrt{155}. \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \vec{b} = 2\vec{i} - \vec{k} \\ & |\vec{a}| = \sqrt{5} \\ & \angle(\vec{a}; \vec{b}) = 30^\circ \\ \hline \text{a) } & \vec{a} \cdot \vec{b} \\ \text{б) } & |2\vec{b} - \vec{a}| \end{aligned}$$

$$\text{a) } \vec{b} = 2 \cdot \vec{i} + 0 \cdot \vec{j} - 1 \cdot \vec{k} = (2; 0; -1);$$

$$|\vec{b}| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5};$$

$$\cos(\angle(\vec{a}; \vec{b})) = \cos 30^\circ = \frac{\sqrt{3}}{2};$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\sqrt{3}}{2} \Leftrightarrow \frac{\vec{a} \cdot \vec{b}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \vec{a} \cdot \vec{b} = \frac{5\sqrt{3}}{2}.$$

$$\text{б) } \vec{a} = (x; y; z); \quad \vec{a} \cdot \vec{b} = 2x + 0 \cdot y - z = 2x - z;$$

$$2x - z = \frac{5\sqrt{3}}{2};$$

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{5} \Rightarrow x^2 + y^2 + z^2 = 5;$$

$$2\vec{b} - \vec{a} = (4 - x; 0 - y; -2 - z);$$

$$\begin{aligned} |2\vec{b} - \vec{a}| &= \sqrt{(4 - x)^2 + (-y)^2 + (-z - 2)^2} = \sqrt{x^2 - 8x + 16 + y^2 + z^2 + 4z + 4} = \\ &= \sqrt{x^2 + y^2 + z^2 - 8x + 4z + 20} = \sqrt{25 - 4(2x - z)} = \sqrt{25 - 4 \cdot \frac{5\sqrt{3}}{2}} = \sqrt{25 - 10\sqrt{3}}. \end{aligned}$$

$$|2\vec{b} - \vec{a}| = \sqrt{25 - 10\sqrt{3}}.$$

③ $\vec{a} \{1; -4; 5\}$
 $\vec{b} \{x; 2; 4\}$
 $\vec{a} \perp \vec{b}$

 $x = ?$

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0 \Leftrightarrow 1 \cdot x - 4 \cdot 2 + 5 \cdot 4 = 0 \Leftrightarrow$$

$$\Leftrightarrow x - 8 + 20 = 0 \Leftrightarrow x = -12.$$

④ $A(1; -3; 3)$
 $B(2; -2; 2)$
 $C(0; 4; -4)$

 $\angle(\vec{BC}; \vec{CA})$

$$\vec{BC} = (0 - 2; 4 - (-2); -4 - 2) = (-2; 6; -6);$$

$$\vec{CA} = (1 - 0; -3 - 4; 3 - (-4)) = (1; -7; 7);$$

$$\vec{BC} \cdot \vec{CA} = -2 \cdot 1 + 6 \cdot (-7) - 6 \cdot 7 = -2 - 42 - 42 = -86;$$

$$|\vec{BC}| = \sqrt{(-2)^2 + 6^2 + (-6)^2} = \sqrt{4 + 36 + 36} = \sqrt{76} = 2\sqrt{19};$$

$$|\vec{CA}| = \sqrt{1^2 + (-7)^2 + 7^2} = \sqrt{99} = 3\sqrt{11};$$

$$\cos(\angle(\vec{BC}; \vec{CA})) = \frac{\vec{BC} \cdot \vec{CA}}{|\vec{BC}| \cdot |\vec{CA}|} = -\frac{86}{2\sqrt{19} \cdot 3\sqrt{11}} = -\frac{43}{3\sqrt{209}};$$

$$\angle(\vec{BC}; \vec{CA}) = \arccos\left(-\frac{43}{3\sqrt{209}}\right) \approx \arccos(-0,99) \approx 172^\circ.$$